

# Comment on “Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity” by V. G. Kogan and J. Schmalian

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The recent paper by V. G. Kogan and J. Schmalian Phys. Rev. B **83**, 054515 (2011) argues that the widely used two-component Ginzburg-Landau (GL) models are not correct, and further concludes that in the regime which is described by a GL theory there could be no disparity in the coherence lengths of two superconducting components. This would in particular imply that (in contrast to  $U(1) \times U(1)$  superconductors), there could be no “type-1.5” superconducting regime in  $U(1)$  multiband systems for any finite interband coupling strength. We point out that these claims are incorrect and based on an erroneous scheme of reduction of a two-component GL theory. *Note added:* below we also attach a separate rejoinder on reply by Kogan and Schmalian.

## I. INTRODUCTION

The recent works Refs. 1 and 2 claim that the two-component Ginzburg-Landau (TCGL) theories can not be used to address any properties of two-component superconductors which involve disparity of density variations, in particular to describe type-1.5 superconducting state where  $\xi_1 < \sqrt{2}\lambda < \xi_2$ <sup>3-18</sup>. Here we point out several crucial errors in the analysis<sup>1,2</sup> thus demonstrating that the following main points in Ref.1 are incorrect,

- attempts to employ the GL functionals, on the one hand, and to assume different length scales, on the other, cannot be justified.
- the idea of 1.5-type superconductivity is not warranted by the GL theory
- $\Delta_1(\mathbf{r}, T)/\Delta_2(\mathbf{r}, T) = \text{const}$ , ... this ratio remains the same at any T in the GL domain

First let us note that Refs. 1 and 2 fail to distinguish between two classes of systems where the type-1.5 state was previously discussed (i)  $U(1) \times U(1)$  and (ii) two-band superconductors where interband coupling explicitly breaks symmetry down to  $U(1)$ . Namely the Refs. 1 and 2 mix up various aspects of physics specific to  $U(1) \times U(1)$  from Ref. 3 (such as the very definition of the coherence lengths) with the different in several respects physics of  $U(1)$  systems. The definitions of coherence lengths and type-1.5 regime in systems with non-zero interband coupling were explicitly discussed in detail in Ref. 6, long before the appearance of Refs. 1 and 2. Thus claims in Refs. 1 and 2 that this coupling was neglected in works on type-1.5 superconductivity are factually incorrect. Note that  $U(1) \times U(1)$  symmetry is also possible in superconductors<sup>19,20</sup> which represents the most straightforward example of systems which cannot be characterized by a single universal GL parameter (physical examples can be found in Refs.3 and 10). However in what follows we focus exclusively on  $U(1)$  two-band superconductors.

Two-component GL (TCGL) model were derived microscopically in Refs. 21–23. However indeed the conditions under which two-component GL expansions for two-band superconductors are formally justified, were not known (to the best of our knowledge) at the time of publication of Refs.1 and 2 but were rigorously established recently<sup>15</sup>. Therefore the aforementioned claim that TCGL expansion is unjustifiable<sup>1,2</sup> is incorrect. In this comment we discuss which incorrect assumptions and technical errors led the authors of Refs.1 and 2 to opposite conclusions.

## II. DEFINITIONS OF THE GL REGIME IN APPLICATION TO $U(1)$ MULTIBAND SYSTEMS.

Let us start with definitions. The Refs. 1 and 2 defines “GL theory” as the free energy proportional to  $\tau^2$  and the modules of the fields varying as  $\tau^{1/2}$  where  $\tau = (1 - T/T_c)$ . Such simplistic definition indeed can be encountered in books on superconductivity which consider simplest single-component systems. However unfortunately such a definition does not work in general. The Ginzburg-Landau theory is a more general concept of a classical field theory description of a system, which in many physical cases does not necessarily appears in the leading order  $\tau$ -expansion. In particular such a definition contradicts all existing literature on multicomponent GL theories in two band superconductors, which in contrast adopts the more general definition of GL expansion of the free energy by powers of gap amplitudes and spatial gradients<sup>21–24</sup>. It should be noted that it most obviously follows from the  $U(1)$  symmetry of two-band superconductors, that the leading order expansion in the parameter  $\tau$  yields a *single* order parameter field characterized by a single coherence length *by construction* (see e.g. a standard textbook Ref.33). The works<sup>21–23</sup>, as well as more recent paper<sup>15</sup> use more general expansion in powers of gradients and amplitudes of the two gap functions  $\Delta_{1,2}$ , which most obviously yields a more complicated temperature dependence, and cannot be expected to be

obtainable in leading order expansion in  $\tau$ . The TCGL expansion for two-band superconductor is thus an example of an expansion in several small parameters (small gaps and gradients). However this fact does not make it unjustifiable as claimed<sup>1</sup>. Indeed recently it was justified on formal grounds for a wide range of parameters<sup>15</sup> (see also remark<sup>26</sup>).

### III. COHERENCE LENGTHS IN TCGL MODEL AND REDUCTION TO THE SINGLE COMPONENT GL THEORY IN THE $T \rightarrow T_c$ LIMIT.

#### A. The reduction argument in Ref. 1

As we discussed above in two band superconductors the broken symmetry is only  $U(1)$ <sup>28</sup> thus, by symmetry, *in the limit*  $T \rightarrow T_c$ , TCGL expansion should be reduced to the conventional text-book single-component GL theory. However the reduction derivation presented in<sup>1</sup> is principally incorrect. The crux of the argument presented in Ref. 1 is that, the TCGL field equations [Eqs. (3,4) in Ref.1]

$$a_1 \Delta_1 + b_1 \Delta_1 |\Delta_1|^2 - \gamma \Delta_2 - K_1 \Pi^2 \Delta_1 = 0 \quad (1)$$

$$a_2 \Delta_2 + b_2 \Delta_2 |\Delta_2|^2 - \gamma \Delta_1 - K_2 \Pi^2 \Delta_2 = 0$$

$$\nabla^2 A - \nabla(\nabla \cdot A)$$

$$+ \frac{16\pi^2}{\phi_0} \sum_{\nu=1,2} K_\nu (\Delta_\nu^* \Pi \Delta_\nu - \Delta_\nu (\Pi \Delta_\nu)^*) = 0 \quad (2)$$

are *generically*, i.e. irrespectively of intercomponent coupling strength  $\gamma$  are well-approximated near  $T_c$  by the simpler system describing condensates with equal coherence length *coupled only by a vector potential* [Eqs. (7) and (8) in Ref.1]

$$\alpha \tau \Delta_1 + \beta_1 \Delta_1 |\Delta_1|^2 - K \Pi^2 \Delta_1 = 0 \quad (3)$$

$$\alpha \tau \Delta_2 + \beta_2 \Delta_2 |\Delta_2|^2 - K \Pi^2 \Delta_2 = 0$$

$$\nabla^2 A - \nabla(\nabla \cdot A)$$

$$+ \frac{16\pi^2}{\phi_0} \sum_{\nu=1,2} K_\nu (\Delta_\nu^* \Pi \Delta_\nu - \Delta_\nu (\Pi \Delta_\nu)^*) = 0, \quad (4)$$

where  $\Pi = \nabla - ie\mathbf{A}$  and the new parameters  $\alpha, K, \beta_\nu$  are related to the coefficients in starting Eqs.(1) and GL parameter  $\tau$ .

Below we present a generic argument that this result and therefore the reduction procedure are incorrect at any temperatures. First we comment that the obtained reduced system of Eqs.(3) contradicts the basic principles of Landau theory. That is, the initial set of equations corresponds to the system with broken  $U(1)$  symmetry. Equations from the second set (3) are coupled only through  $\mathbf{A}$  and thus corresponds to independently conserved condensates. Thus Eqs.(3) are the field equations corresponding to a free energy functional with spontaneously broken  $U(1) \times U(1)$  symmetry. Note that the

interband Josephson coupling in the initial set of equations breaks the symmetry of the system down to  $U(1)$  symmetry, but no phase locking terms are present in the reduced system of equations. Therefore the reduced theory fails to account to this effect and is wrong already on symmetry grounds. Furthermore Landau theory for  $U(1)$  systems dictates that there is only one diverging coherence length *in the limit*  $T \rightarrow T_c$  associated with a single complex field (but not two degenerate coherence lengths associated with two fields coupled by vector potential only).

#### B. Coherence lengths in two-band superconductors

The work 1 (and the recent follow up quoted therein 25) claim that the gap fields  $\Delta_{1,2}$  have two independently diverging in the limit  $T \rightarrow T_c$  coherence lengths. They claim that coherence lengths are directly attributed  $\Delta_{1,2}$  and that they become degenerate in some domain, which these authors call “GL domain” at small  $\tau$ , where the system is claimed to be described by two equations coupled only by vector potential.

Such incorrect conclusion regarding the evolution of the length scales in the  $T \rightarrow T_c$  limit is based on misunderstanding of how coherence lengths are defined in two-band systems. The erroneous claim that two coherence lengths are attributed directly to  $\Delta_{1,2}$  and that they become identical near  $T_c$  originates in the attempt<sup>2</sup> to assess coherence lengths through a comparison of the gap function profiles in the 1D boundary problem. From the observation that the overall profiles become identical in the  $T \rightarrow T_c$  limit the authors of Refs.1 and 2 concluded that the two gap functions  $\Delta_{1,2}$  are characterized by the similar coherence lengths. Such approach is technically incorrect because one cannot extract information of coherence length from the naive inspection of an overall density profiles in a nonlinear theory. Instead the correct analysis of coherence lengths in two-band superconductor requires an accurate consideration of the asymptotic solutions of linearized field equations for the gap functions<sup>10,14</sup>. As shown<sup>10,14,15</sup> in the wide range of parameters for finite interband Josephson coupling there exist two asymptotical normal modes with different coherence lengths (or inverse masses of the normal modes). The two distinct coherence lengths appear as a result of hybridization of the superconducting gap fields, and *cannot* be directly attributed to the  $\Delta_{1,2}$  fields at any finite Josephson coupling and at any temperature. Instead normal modes are associated with linear combinations of  $\Delta_{1,2}$  and thus coherence lengths are hybridized<sup>6,10,14,15</sup>. Moreover one of the two coherence lengths does not diverge in the limit  $\tau \rightarrow 0$ . In fact the disparity between coherence lengths grows rather than shrinks in that limit<sup>14,15</sup>. This in particular means that type-1.5 regime in two-band superconductors cannot have anything in common with a two-component counterpart of Bogomolnyi regime of single-component superconductors

with  $\kappa \approx 1/\sqrt{2}$  (see e.g. review<sup>27</sup>).

The overall gap function profiles are determined by nonlinearities and thus not only by masses of the normal modes but also by their amplitudes<sup>15</sup>. Therefore it is not possible to extract the information about the coherence lengths just analyzing the overall profile of the gap functions, like was attempted in Ref<sup>2</sup>. The correct reduction of TCGL model to single-component GL theory takes place because in the limit  $T \rightarrow T_c$  the mode with a non-diverging coherence length loses its *amplitude*<sup>15</sup>, but not because two coherence lengths gradually become degenerate.

#### IV. MISCONCEPTIONS

i For unclear reasons, the Ref. 1 criticises previous works on type-1.5 superconductivity for “*assumption of two different penetration lengths  $\lambda_{1,2}$* ”. We are not aware of any papers on two-band superconductivity where such assumptions were made. As far as we know the notations  $\lambda_{1,2}$  were used in literature on type-1.5 superconductivity only as characteristic constants, parameterizing GL free energy while the physical magnetic field penetration length was always determined self-consistently.

ii In contrast to what was attributed to us in Ref.2 no assumptions of having zero interband coupling but equal  $T_c$  for all components in two-band systems were made<sup>3,6,10,14</sup>.

iii The work Ref.1 claims that it is not possible to obtain fractional vortices in multicomponent superconductors discussed in the Refs.<sup>31</sup>. Obviously *in the limit*  $T \rightarrow T_c$  in two-band systems fractional vortex excitations are suppressed. However fractional vortices can exist in  $U(1) \times U(1)$  systems and in two-band  $U(1)$  theories at finite- $\tau$ . The authors of Ref.1 missed that the papers in Ref.31 deal with fractional vortex solutions not in the  $T \rightarrow T_c$  limit and not even in the GL model but exclusively in the London theory. In fact in a GL model the fractional vortex solutions are quite different (see corresponding discussion for  $U(1) \times U(1)$  systems in Ref.30). Moreover Refs. 31 primarily focuses on the  $U(1) \times U(1)$  systems. Thus the results in Ref.31 are entirely unrelated to the arguments on  $T \rightarrow T_c$  limit in two-band systems. However we mention that occurrence of fractional vortices in two-component GL models with intercomponent Josephson coupling in mesoscopic samples was investigated by other groups<sup>32</sup>.

iiii The work<sup>1</sup> also contains mutually exclusive claims. On one hand from the incorrect derivation of Eqs.(3) it would follow that in the limit  $T \rightarrow T_c$  the  $U(1)$  TCGL theory is reduced to the  $U(1) \times U(1)$  theory when the gap functions are coupled only by the vector potential (and not by phase-locking terms). On the other hand Ref.<sup>1</sup> claims that in two-band superconductors the GL theory can only describe the gap functions having the same phase. If the former of these claims contradicts the basic principles of Landau theory (see the discus-

sion above), the latter statement also yields unreasonable conclusions negating for example the existence of the phase difference excitations<sup>29</sup>. At finite- $\tau$  when two-band GL theory is well justified, the appearance of the gradients of the phase difference between component at finite  $\tau$  is in fact a quite generic effect<sup>11</sup> because the mass of the phase-difference mode does not diverge in the  $T \rightarrow T_c$  limit<sup>15</sup>.

#### V. CONCLUSIONS

We discussed the errors in the treatment of  $T \rightarrow T_c$  limit in two-component superconductors in Ref.<sup>1</sup>, which led to the incorrect (at any temperatures) system of field equations (3) for the gap fields and incorrect conclusions on the behavior of coherence lengths. We also pointed out that contrary to the claims in Ref.1 TCGL expansion is justified and can be used to describe systems with disparity in coherence lengths as was demonstrated on formal grounds in<sup>15</sup> and does allow type-1.5 regimes.

*Note added:* below we also attach a separate rejoinder on reply by Kogan and Schmalian.

#### VI. ACKNOWLEDGEMENTS

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# Rejoinder on reply by V. G. Kogan and J. Schmalian

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In their reply<sup>34</sup> to our recent comment<sup>35</sup>, Kogan and Schmalian did not refute or, indeed, discuss the main points of criticism in the comment. Unfortunately they instead advance new incorrect claims regarding two-band and type-1.5 superconductivity. The main purpose of this rejoinder is to discuss these new incorrect claims.

(i) First we would like to emphasize that in our comment we observed that the attempted GL construction in<sup>36</sup> is incorrect *at all temperatures* and contradicts the basic (symmetry-based) principles of Landau theory. Thus it is not clear to us why in their reply the authors Ref.34 would assert that we “share” the opinions espoused in<sup>36</sup>. Perhaps we did not make this point clear enough. The standard understanding in the field has always been indeed that a GL expansion to leading order in  $\tau$  gives a single GL equation. This elementary mean-field theory fact<sup>47</sup> follows from simple symmetry considerations. However, since the mid-1960s it has also been known that, in principle, one can obtain two-component GL equations from a multi-parameter expansion in two-band systems<sup>45</sup> (see also remark<sup>38</sup>.) As we pointed out in our comment<sup>35</sup>, the picture presented by Kogan and Schmalian in<sup>36</sup> strongly disagrees with the basic principles of GL theory: instead of obtaining a standard single-component GL equation the authors of Ref. 36 obtained a *system* of GL equations in the leading order in  $\tau = (1 - T/T_c)$ . This system of GL equations corresponds to  $U(1) \times U(1)$  broken symmetry which is principally incorrect for two-band superconductors with  $U(1)$  broken symmetry.

The correct derivation of a single GL equation in the limit  $\tau \rightarrow 0$ , and a different system of two-component GL equations for small but nonzero  $\tau$ , are discussed in Ref.<sup>43</sup>. The authors of<sup>34</sup> did not refute the criticism in our comment that, in two-band superconductors, the two-component GL equation with different coherence lengths is rigorously obtained by a multiparameter expansion, and is not an expansion in a single parameter  $\tau$ .

(ii) The authors of reply<sup>34</sup> incorrectly speculate how coherence lengths behave near  $T_c$  in two-band superconductors, using their spurious form of the GL equations. They claim that the difference in coherence lengths in two-band superconductors near  $T_c$  cannot be larger than  $\tau^2$ . This erroneous conclusion is merely a consequence of the incorrect  $U(1) \times U(1)$  form of their “leading order in  $\tau$ ” GL functional. It is possible to verify that it is indeed wrong by a comparison to the known (at all temperatures) behavior of coherence lengths in a full microscopic theory. The Figure 1 shows the actual behav-

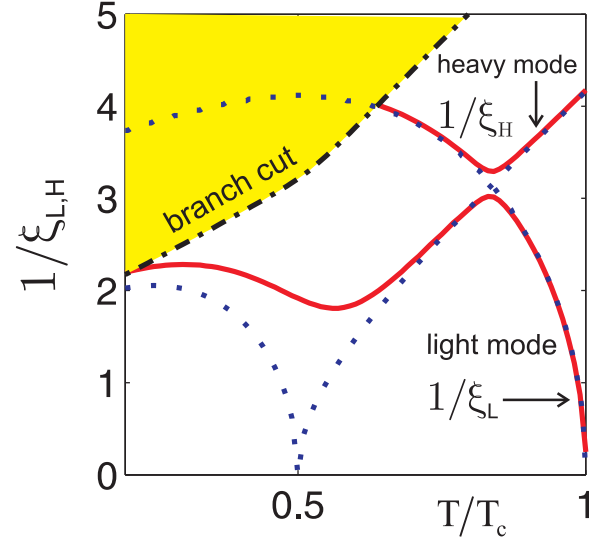


FIG. 1. Comparison of the inverse coherence lengths  $\xi_{L,H}^{-1}(T)$  of a  $U(1) \times U(1)$  model (dotted blue lines) and a two-band  $U(1)$  model with interband pairing (red lines). The  $U(1) \times U(1)$  model has  $T_{c1} = 1$  and  $T_{c2} = 0.5$ . In the  $U(1) \times U(1)$  case, one of the coherence lengths diverges at  $T_{c2} = 0.5$ . Above this temperature, for this component we plot the coherence length associated with superconducting fluctuations in the normal state (around zero vacuum expectation value of superconducting condensate for that component). It indeed diminishes with increasing temperature. When a very small interband coupling is added in this example, for obvious reasons, it does not affect dramatically the characteristic length scales. It leads only to a small hybridization of coherence lengths in this case<sup>39,40,42,43</sup>. However (i) the interband Josephson coupling removes a phase transition and a divergence of coherence length at  $T_{c2}$  and (ii) it gives both components nonzero expectation value above  $T_{c2}$  without dramatically affecting the characteristic length scales. Thus there is growing disparity of coherence length near  $T_c$  in such two-band superconductors. Note that these coherence lengths are associated with linear combinations of the gap fields. If one plots cores in the individual gap fields or GL theory fields  $\psi_{1,2}$ , then, because of nonlinear effects, the *overall density profile of vortex cores in  $\psi_{1,2}$  will become increasingly similar near  $T_c$  despite the divergence in the disparity of coherence lengths*<sup>42,43</sup>. This is because near  $T_c$  there is less and less density associated with the mode which has short coherence length. This effect was mistaken for being a signature that coherence lengths become similar in the erroneous analysis in<sup>37</sup>.

ior of two coherence lengths  $\xi_{L,H}$  calculated in a microscopic two-band Eilenberger theory<sup>42,43</sup> with interband



coupling properly taken into account. Clearly the disparity of  $\xi_L$  and  $\xi_H$  near  $T_c$  does not disappear but instead diverges because only one coherence length  $\xi_L$  diverges in the limit  $T \rightarrow T_c$ . This behavior is also perfectly captured by the two-component GL theory<sup>43</sup>.

Thus the behavior of coherence lengths proposed in the reply<sup>34</sup>, not only is wrong on symmetry grounds but also contradicts the results of microscopic theory<sup>42,43</sup> which does not rely on any form of GL expansion.

(iii) Based on the erroneous behavior of coherence lengths the authors of reply<sup>34</sup> further advance an obviously incorrect claim that one should have  $\kappa_1$  and  $\kappa_2$  close to  $1/\sqrt{2}$  in order to have type-1.5 behavior near  $T_c$ . (We do not use the notations  $\kappa_1$  and  $\kappa_2$ , which also were not defined in<sup>34</sup>; we assume that they meant  $\kappa_1 = \lambda/\xi_1$  and  $\kappa_2 = \lambda/\xi_2$ .) The authors of<sup>34</sup> then assert that it is related to “the situation studied by Essmanns group”, and “the vortex attraction in small fields for  $\kappa$  close to  $1/\sqrt{2}$  was not a sufficient reason for declaring a new type of superconductivity”. In fact Essmann’s group studied a *single*-component type-2 superconductor which is close to the Bogomolnyi limit  $\kappa \approx 1/\sqrt{2}$ . These statements are obviously wrong since

(a.) In two-band systems the disparity between  $\xi_1$  and  $\xi_2$  *diverges* near  $T_c$  in GL theory. Thus the type-1.5 regime has nothing to do with any kind of two-component counterpart of the Bogomolnyi ( $\kappa \approx 1/\sqrt{2}$ ) regime.

(b.) As emphasized in our works, in single-component type-2 systems indeed there is a well known effect of inter-vortex attraction due to microscopic corrections in single-component theory with  $\kappa \approx 1/\sqrt{2}$ . In that regime, at the level of GL theory vortices almost do not interact and small microscopic corrections become important. The physics of microscopic corrections in single-component system is not universal (i.e. it is not based on fundamental length scales but is a consequence of some particular, e.g. weakly coupled BCS theory). Indeed by itself it does not constitute a new regime of superconductivity since Nb studied by Essmann et.al. is still characterized by a single ratio of two fundamental length scales  $\xi$  and  $\lambda$  which categorizes it as a type-2 superconductor.

(c.) The authors of Ref.34 miss the fact that attractive intervortex interaction is not a defining property of “type-1.5 regime”. All the physics of type-1.5 super-

conductivity is about multi-component superconducting states with  $\xi_1 < \sqrt{2}\lambda < \xi_2$ , and thus about the coexistence of competing type-1 and type-2 behaviors of *two* or more components (see e.g. the review<sup>44</sup>).

Finally we do not agree with the assertion in<sup>34</sup> that the proceeding contributions of Brandt and Das quoted therein provide an accurate review of the single-component  $\kappa \approx 1/\sqrt{2}$  regime, and unfortunately we had to comment on this elsewhere<sup>48</sup>.

(iv) In the reply<sup>34</sup> the authors claim “*in the statement that in “the GL domain” the phases of two order parameters must be the same modulo  $\pi$ , we had to stress that this is true for the minimum energy state (equilibrium). Otherwise, a reader may deduce that we negate the existence of the Leggett mode as a possible excitation.*”

First we note that this contradicts what was stated in the original paper by Kogan and Schmalian<sup>36</sup>. Namely in Ref.<sup>36</sup> the argument that “there is only one phase” was used to claim that there are no fractional vortex excitations. These vortices are nothing but phase difference excitations.

Second, the above-quoted statement is self-contradictory. On one hand both in the original paper<sup>36</sup> and in the reply<sup>34</sup> the authors claim the phases of the gap functions are locked in what they call the “GL domain”. But on the other hand this statement directly contradicts the system of equations which they write (eqs. (7),(8) in<sup>36</sup>). That system indeed does not have any phase locking terms at all. Allowing phase difference variations in the model<sup>36</sup> does not yield a Leggett mode in contrast to what was claimed in<sup>34</sup>. Namely the system of two equations for the gap functions in<sup>36</sup> does not contain the interband coupling terms. The condensates are independently conserved and thus such a system does not support the Leggett mode which appears due to interband tunneling of Cooper pairs in two-band superconductors. Instead in the incorrect GL construction in ref.<sup>36</sup> the phase difference mode is an unphysical Goldstone mode, existence of which is forbidden by symmetry in two-band superconductors. Thus the authors claim physical effects which are in direct contradiction with the equations they write (phase locking effect in the absence of any phase locking terms or Leggett mode in the absence of any interband tunneling).

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- <sup>27</sup> In two-band systems where intercomponent coupling breaks symmetry down to  $U(1)$ , by symmetry, it is obvious that at mean-field level, GL theory should reduce to a single-component one *in the limit*  $\tau \rightarrow 0$ . However the limiting  $\tau \rightarrow 0$  analysis unfortunately does not have any physical significance in a generic two-band system. Essentially for any finite  $\tau$  it misses the qualitative effect of the presence of two coherence lengths for the superconducting density variation realized by "heavy" and "light" asymptotical modes<sup>14,15</sup>. The type-1.5 regime in such systems is associated with the interplay of two density scales and the magnetic field penetration depth. Furthermore as shown in microscopic calculations the masses of the fields (inverse length scales) in two-band models can change rapidly and in a non-trivial way with decreasing temperature<sup>14,15</sup>. Thus a limiting  $\tau \rightarrow 0$  analysis in multiband system, in general, cannot give even an approximate physical picture even at very small  $\tau$ .
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